RESEARCH DEPARTMENT

COLOUR TELEVISION: THE REDUCTION OF COLOUR SATURATION RESULTING FROM VESTIGIAL-SIDEBAND RECEPTION

Report No. G-065

(1955/38)

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SUMMARY

This report deals with one form of distortion introduced by the reception of colour television using a vestigial-sideband system. In a system such as that of the N.T.S.C. modified to British standards, incoherent detection results in an increase of the luminance signal and a reduction in the chrominance signal. The effect with colour receivers is to reduce the saturation of colours without changing their hues. An extreme case is that of a fully saturated blue signal of maximum amplitude for which the luminance component is increased by 75% and the saturation consequently reduced by 46%. In the case of a fully saturated blue signal of 75% full amplitude, the luminance component is increased by 56% and the saturation reduced by 38%.

The effect with monochrome receivers is to increase the luminance of those parts of the received picture where the original colour is of high saturation; this effect, which cannot be eliminated by using a "notch" filter in the video stage to reduce the chrominance signal amplitude, may in some circumstances have desirable consequences.

1. INTRODUCTION.

The distortion of monochrome television signals resulting from vestigial-sideband reception has been analysed[1,2,6]. In practice the distortion is not serious when allowance is made for the saving of bandwidth, because the lower-frequency components of the modulation are in fact received by a double-sideband system and the higher-frequency components tend to be small. This is not the case for a colour television system such as, for instance, the N.T.S.C.[3] system, in which the entire colour signal is transmitted as a single sideband. For such a system the effect of single sideband reception warrants re-examination.

2. THE DETECTED OUTPUT FROM A SINGLE-SIDEBAND RECEIVER.

In the case of the colour television system to be considered the colour signal is transmitted as a single sideband. Since only the distortion associated with the colour signal is to be examined, attention will be confined to the single-sideband system.

The frequency components resulting from the modulation of a carrier, $\sin\omega t$, by a waveform $\{1 + m\cos(pt + \phi)\}$ are:

i. The carrier wave, $\sin\omega t$.

ii. A low frequency sideband $\frac{m}{2}\sin[(\omega-p)t-\phi]$.

iii. A high frequency sideband $\frac{m}{2}\sin[(\omega+p)t+\phi]$.

If, as a result of passing the signal through a filter network, one sideband is removed before transmission, the transmitted waveform can be expressed as,

$$f(t) = E_R \sin(\omega t + \alpha)$$

where

$$E_R = \sqrt{1+\frac{m^2}{4}+m\cos\theta}$$

$$\alpha = \arcsin \frac{m}{2}\cdot\frac{\sin\theta}{E_R}$$

$$\theta = \pm pt \pm \phi$$

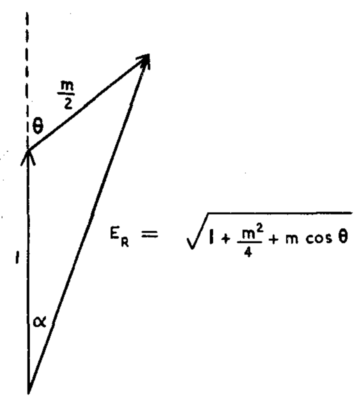

Fig. 1

The positive or negative signs in the last expression correspond to the removal of the low or high-frequency sidebands respectively.

The envelope (E_R) of this waveform has a mean value greater than unity. In other words, the removal of one sideband of a double sideband system results in an increase in the mean value of the envelope. If the signal is rectified by a linear incoherent detector the output waveform can be expressed as,

$$E_R = \left.\begin{array}{l} a_0 + a_1\cos\theta + a_2\cos 2\theta + \ldots\ldots \\ \quad + b_1\sin\theta + b_2\sin 2\theta + \ldots\ldots \end{array}\right\} \tag{1}$$

where

$$a_0 \text{(the mean value of } E_R\text{)} = \frac{1}{\pi}\int_0^{\pi} E_R\, d\theta. \tag{2}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} E_R \cos n\theta,\, d\theta \tag{3}$$

$$\text{and } b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} E_R \sin n\theta,\, d\theta \tag{4}$$

In the case to be considered the components $a_n \cos n\theta$ and $b_n \sin n\theta$ for $n > 1$ will be removed as a result of the limited bandwidth of the receiver. Substituting

$$k = \frac{\sqrt{2m}}{1+\frac{m}{2}}$$

and $\psi = \frac{\theta}{2}$ the values of a_0, a_1 and b_1 are given by,

$$a_0 = (1+\frac{m}{2})\frac{2}{\pi}\int_0^{\pi/2}\sqrt{1-k^2\sin^2\psi}\,d\psi$$

$$= (1+\frac{m}{2})\frac{2}{\pi}\,E(\frac{\pi}{2},k) \qquad (5)$$

where $E(\frac{\pi}{2},k)$ is the complete elliptic integral of the second kind.*

[When $k^2 < 1$, $E(\frac{\pi}{2},k) = \frac{\pi}{2}\{1-(\frac{1}{2})^2 k^2 - (\frac{1.3}{2.4})^2\frac{k^4}{3} - (\frac{1.3.5.}{2.4.6.})^2\frac{k^6}{5}\ldots\}$;

when $k = 1{\cdot}0$, $E(\frac{\pi}{2},k) = 1{\cdot}0$]

$$a_1^{**} = (1+\frac{m}{2})\frac{2}{\pi}\int_0^{\pi/2}(1-2\sin^2\psi)\sqrt{1-k^2\sin^2\psi}\,d\psi$$

$$= 2a_0 - (1+\frac{m}{2})\,\frac{4}{\pi}B(\frac{3}{2},\frac{1}{2})F(\frac{3}{2},\frac{1}{2},2;\,k^2) \qquad (6)$$

where $B(a,b) = \frac{\Gamma(a).\Gamma(b)}{\Gamma(a+b)}$

is the Beta Function and

$$F(\alpha,\beta,\gamma;\,x) = 1+\frac{\alpha\beta}{1!\gamma}x+\frac{\alpha(\alpha+1)\beta(\beta+1)}{2!\,\gamma(\gamma+1)}x^2+\frac{\alpha(\alpha+1)(\alpha+2)\beta(\beta+1)(\beta+2)}{3!\,\gamma(\gamma+1)(\gamma+2)}x^3+\ldots$$

is the Gaussian hypergeometric function

$$b_1 = (1+\frac{m}{2})\frac{2}{\pi}\int_{-\pi/2}^{\pi/2}(\sin\psi\cos\psi)\sqrt{1-k^2\sin^2\psi}\,d\psi = 0. \qquad (7)$$

As the value of $b_1 = 0$ the fundamental component of the detected output waveform has the same phase as when double sidebands are present. As the envelope of E_R is symmetrical about $\theta = 0$ the components $b_n \sin n\theta$ in (1) will all be zero. Fig. 2† shows a_0 and a_1 plotted as functions of m. If $m \ll 1$, a_0 is equal to

$$(1+\frac{m^2}{16})\;;$$

a_1 is always less than $\frac{m}{2}$, but approaches this value as $m \to 0$. It is of interest to note that the power transmitted in a single-sideband system is

*Tables of these integrals are to be found in B.O. Pierce: A Short Table of Integrals, or Jahnke and Emde (1938) p. 69.

**Integraltafel, Gröbner und Hofreiter, II, 331, 93.

† The same result, expressed in a different mathematical form, has been given by F.M. Colebrook. "The Frequency Analysis of the Heterodyne Envelope", Wireless Engineer, April 1932.

$$(1+\frac{m^2}{4})$$

times the carrier power, but the power in the detected wave is redistributed so that the d.c. power component is greater than

$$(1+\frac{m^2}{8})$$

while the modulation power component is less than $\frac{m^2}{8}$.

In other words, in a single sideband system the useful component of the modulation contains less than half the power of the transmitted single sideband; the remainder is effectively transferred to the carrier if an incoherent detector is used. On the other hand it can be shown that if a coherent detector is used, half the modulation power is lost in the detector.

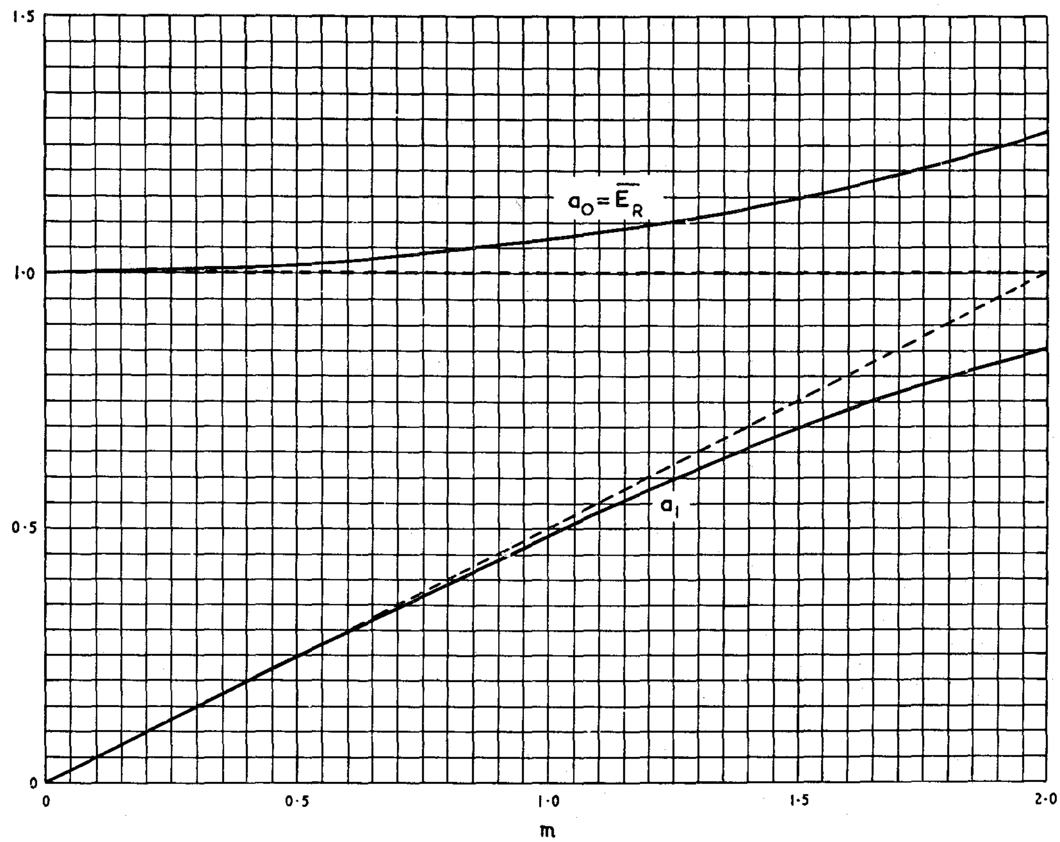

Fig. 2

3. RECEPTION USING A COLOUR RECEIVER.

3.1. N.T.S.C. System Modified to British Standards.

The effect of single-sideband reception of the N.T.S.C.[3] colour television system, modified to suit British standards, will next be examined in the light of the

foregoing analysis. In this system the picture information is divided into three component parts. One is the luminance or brightness of the picture element, which is transmitted in the same manner as for monochrome television. The other two components, which together form the chrominance information, are the hue and the saturation. The chrominance information is conveyed by a sine wave of approximately 2·66 Mc/s which is superimposed on the luminance signal; the amplitude of this sine wave relative to that of the luminance signal determines the saturation whilst the phase angle, relative to that of a reference sine wave determines the hue. The reference signal consists of a short "burst" of the same frequency radiated between the line synchronising pulses and the line picture waveforms.

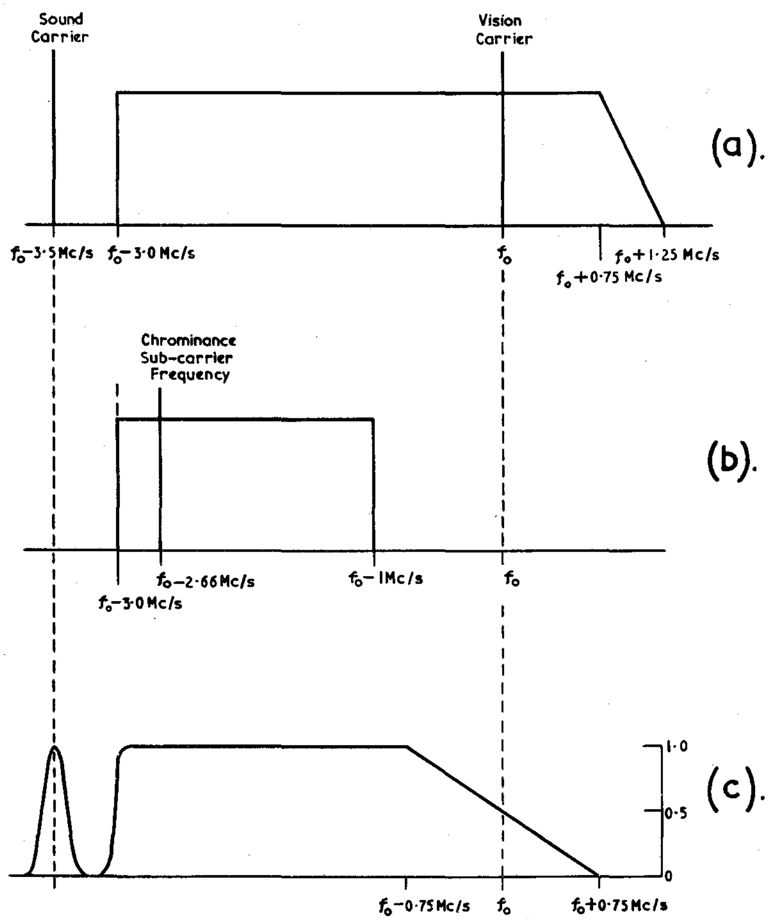

Fig. 3 - (a) Transmission characteristic for luminance signals
(b) Transmission characteristic for chrominance signals
(c) Receiver characteristic

Fig. 3 shows the transmission characteristic for the luminance and chrominance signals, and the receiver response for the composite signal. It will be seen that the receiver response at the vision carrier frequency is reduced by 6 dB relative to that for the colour signal. This is equivalent to doubling the amplitude of the sideband in Fig. 1, i.e., doubling the value of m in Fig. 2; for example, m = 2 for 100% modulation of the carrier.

The complete colour television picture waveform can be expressed as

$$E_m = E_y' + E_c' \cos(\omega_s t + \phi) \tag{8}$$

where E_y' = the luminance signal, is equal to $0{\cdot}3\,E_R' + 0{\cdot}59\,E_G' + 0{\cdot}11\,E_B'$

E_R', E_G', E_B' = the gamma corrected red, green and blue components of the colour signal.

$$E_c' = \sqrt{E_I^2 + E_Q^2}$$

$$\phi = -\arctan\frac{E_Q}{E_I}$$

$$E_Q = 0{\cdot}877\,[\,0{\cdot}562(E_B' - E_y')\cos 33^\circ + (E_R' - E_y')\sin 33^\circ]$$

$$E_I = 0{\cdot}877\,[\,(E_B' - E_y')\cos 33^\circ + 0{\cdot}562(E_R' - E_y')\sin 33^\circ]$$

ω_s = $2\pi.\,2{\cdot}6578125.10^6$ radians per second.

The angle ϕ is the parameter which determines the hue, and as shown in Section 2 is not altered by single-sideband reception. We will therefore confine our attention to E_y' and E_c'; Fig. 4 shows the relative amplitude of these components for six saturated colours, and also for black and white. The complete modulation for a colour transmission consists of E_y' and E_c' added to a synchronising waveform in such a way that the final amplitude range between zero and 30% is used for synchronising pulses, and the range above 35% is used for the picture information*. For this division the value for peak white is taken as 100% although, in practice, some saturated colour signals will exceed this value. The resulting waveform is shown in Fig. 5 which represents one line of a picture consisting of vertical bars of colour.

If we consider the modulation waveform for a steady-state transmission of saturated red signal of maximum amplitude we see from Fig. 5 that it can be expressed as,

$$E_m = 0{\cdot}545(1 + 0{\cdot}75\cos pt)$$

The waveform is shown in Fig. 6(a) and the resultant envelope for the single-sideband transmission is shown in Fig. 6(b). A conventional receiver will attenuate the carrier component by 6 dB relative to the sideband, and the envelope at the detector is shown in Fig. 6(c); this waveform has been scaled up in the ratio 2 to 1 for comparison with other figures. The dotted curve in Fig. 6(c) shows the output from a linear incoherent detector after the removal of all components having frequencies higher than the fundamental,

$$\frac{p}{2\pi}$$

The mean value (a_0) is seen by reference to Fig. 2 ($m = 2 \times 0{\cdot}75$) to be equal to 1·15,

*The colour synchronising "burst" need not here concern us.

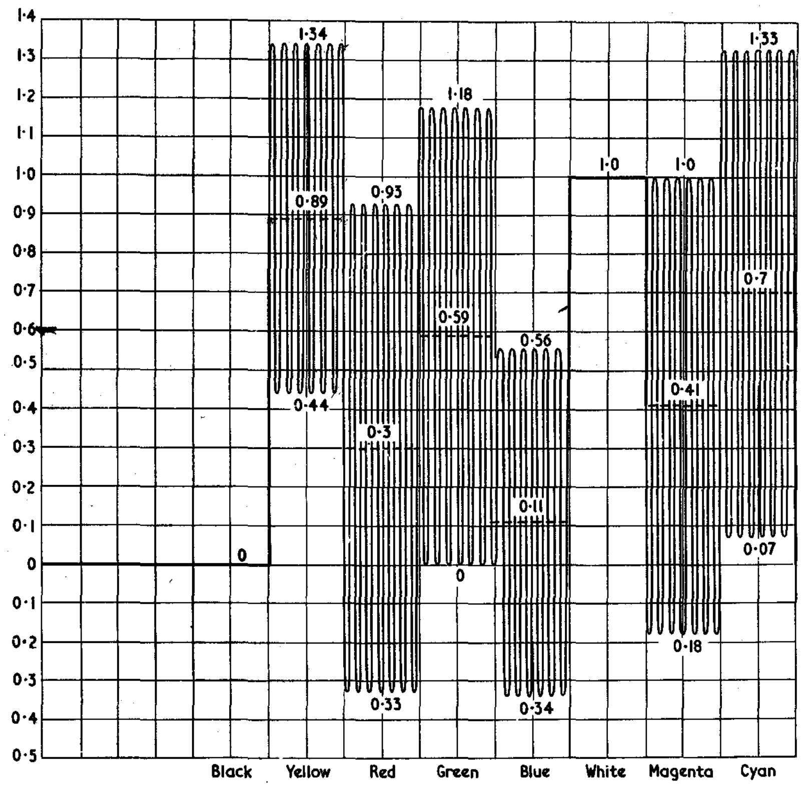

Fig. 4 - Modulation waveform without sync. pulses and colour burst (peak white = 1·0)

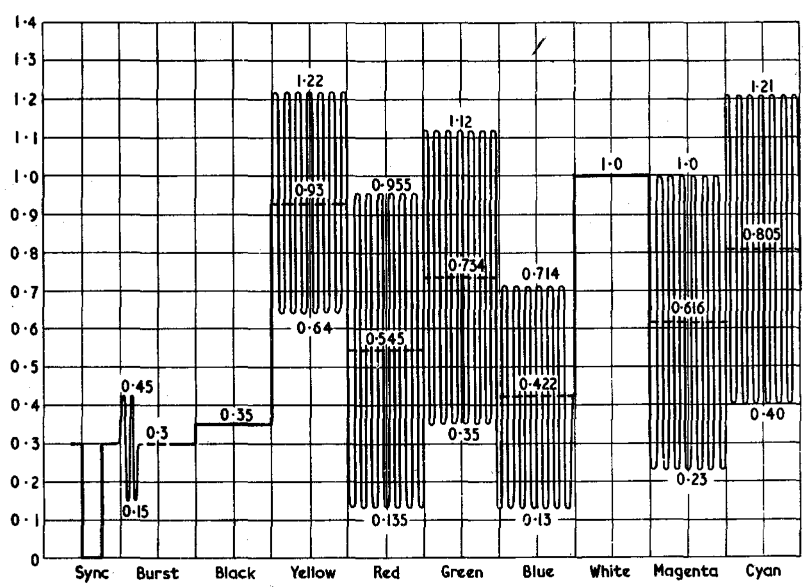

Fig. 5 - Modulation waveform for saturated colours with sync. pulses and colour burst (peak white = 1·0)

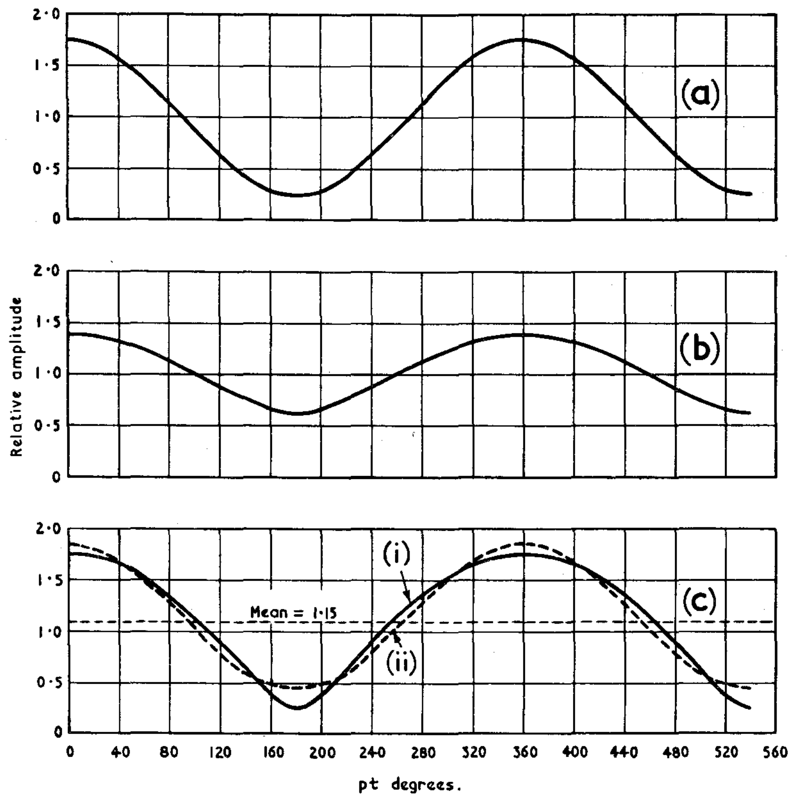

Fig. 6 - (a) Modulation waveform for saturated red $(1 + 0{\cdot}75 \cos pt)$

(b) Envelope of s.s.b. transmission $\sqrt{1 + \frac{(0{\cdot}75)^2}{4} + 0{\cdot}75 \cos pt}$

(c) (i) Detected output of s.s.b. receiver (6 dB carrier loss),

$$\sqrt{1 + \frac{(1{\cdot}5)^2}{4} + 1{\cdot}5 \cos pt}$$

(ii) After removal of harmonics of $p, (1{\cdot}15 + 0{\cdot}7 \cos pt)$

while the value of a_1 is 0·7. Thus the original modulation 0·545 $(1 + 0{\cdot}75 \cos pt)$ has been changed to 0·545 $(1{\cdot}15 + 0{\cdot}7 \cos pt)$. The synchronising portion of the waveform will be effectively removed in the receiver as a result of the receiver brightness being adjusted so that only those portions of the waveform which are more positive than 0·35 will produce perceptible luminance. This will leave the luminance component increased from 0·195 to 0·277, an increase of 42%. Table 1 shows the relative values of E_y' and E_c' in the modulation waveform and in the receiver detected output, for a range of saturated colours of maximum amplitude. The changes in the values of the chrominance signals are small, but the changes in the luminance signals are more serious.

The values shown in Table 1 are for fully saturated colours at maximum amplitude. This condition is not likely to occur frequently, and a more practical case to consider is that of saturated colours having 75% of maximum amplitude since in this case, which coincides with the American FCC Specification figures, no signal exceeds the amplitude of peak white. Table 2 shows that the distortion in this case

TABLE 1

Saturated Colours 100% Amplitude using Experimental British System of Modulation

Colour	Before Transmission		After Detection		Increase of E'_y, %	Reduction of E'_c, %
	E'_y	E'_c	E'_y	E'_c		
Black	0	0	0	0	0	0
Yellow	0·89	0·45	0·96	0·44	8	2
Red	0·3	0·63	0·43	0·58	42	8
Green	0·59	0·59	0·69	0·56	16	6
Blue	0·11	0·45	0·19	0·42	75	7
White	1·0	0	1·0	0	0	0
Magenta	0·41	0·59	0·5	0·55	22	7
Cyan	0·7	0·63	0·81	0·59	15	8

TABLE 2

Saturated Colours 75% Amplitude using Experimental British System of Modulation

Colour	Before Transmission		After Detection		Increase of E'_y, %	Reduction of E'_c, %
	E'_y	E'_c	E'_y	E'_c		
Black	0	0	0	0	0	0
Yellow	0·66	0·34	0·68	0·34	3	1
Red	0·22	0·47	0·3	0·45	36	5
Green	0·44	0·44	0·49	0·43	11	3
Blue	0·09	0·34	0·14	0·33	56	3
White	0·75	0	0·75	0	0	0
Magenta	0·31	0·44	0·37	0·42	19	4·5
Cyan	0·53	0·47	0·6	0·44	13	6

is less serious. It will also be less for desaturated colours which have a smaller ratio of chrominance signal amplitude to luminance signal amplitude.

If the receiver were designed to maintain the relative amplitudes of the transmitted carrier and sidebands right through to the detector, instead of introducing 6 dB attenuation at the carrier frequency, the modulation depth would be halved and the distortion considerably reduced. Table 2 shows that with a conventional receiver a fully saturated blue signal having 75% of maximum amplitude has its luminance component increased by 56%. If the receiver had uniform gain for carrier and sideband the luminance increase would only be 14% and the chrominance reduction 0·5%. The increased d.c. and low frequency response of such a receiver could be corrected in the video stages.

The effect of the increase in the luminance component in all the cases considered above is exaggerated because the screen brightness of the receiver cathode ray tube is approximately proportional to $(E'_y)^{\gamma}$ where γ is 2·5. This means that the effective luminance of a saturated blue signal of maximum amplitude will be

increased by a factor of 4; similarly the luminance of a red signal will be increased by a factor of 2·4.

3.2. N.T.S.C. with American Standards*.

Television carriers in the U.S.A. are modulated in such a manner that an increase of carrier amplitude represents a reduction of picture brightness. In this case the apparent increase of carrier amplitude resulting from single sideband reception of saturated colours will cause a reduction in the luminance signal amplitude; Table 3 shows the values of E'_y and E'_c before and after reception. Comparison with Table 2 shows that single-sideband reception introduces less distortion of the luminance signal when the American system of modulation is used. With this system, however, the distortion results in an increase of saturation.

TABLE 3

Saturated Colours 75% Amplitude using American System of Modulation

Colour	Before Transmission		After Detection		Reduction of E'_y, %	Reduction of E'_c, %
	E'_y	E'_c	E'_y	E'_c		
Black	0	0	0	0	0	0
Yellow	0·66	0·34	0·61	0·33	8	3
Red	0·22	0·47	0·16	0·46	27	2
Green	0·44	0·44	0·38	0·43	14	2
Blue	0·09	0·34	0·06	0·34	33	1
White	0·75	0	0·75	0	0	0
Magenta	0·31	0·44	0·25	0·43	19	2
Cyan	0·53	0·47	0·45	0·45	15	4

4. RECEPTION USING A MONOCHROME RECEIVER.

The N.T.S.C. system does not produce a true panchromatic monochrome picture[4]; each colour is reproduced in a shade of grey which is insufficiently bright. The increase in the luminance signal which occurs with the British system of modulation therefore has the effect of improving the grey scale of the monochrome picture.

If the receiver has a "notch" filter to remove the chrominance signal in the video stage the brightness of the picture will be proportional to $(E'_y)^\gamma$. Table 4 shows the calculated values of $(E'_y)^\gamma$ for saturated primary and complementary colours having 75% of maximum amplitude. The value of γ has been taken as 2·5. Table 4 gives the results for a single-sideband receiver and a double-sideband receiver, and also the true panchromatic values. Single-sideband monochrome reception of a colour signal is more nearly panchromatic than double-sideband reception.

*Since this report was written the author's attention has been drawn to an article by G.L. Fredenhall and W.C. Morrison, "Effect of Transmitter Characteristics on N.T.S.C. Color Television Signals", Proc. I.R.E., Vol. 42, No. 1, January 1954. This article deals with the effect of single-sideband transmission as well as other effects peculiar to the N.T.S.C. system.

TABLE 4

Effective Relative Luminances for Receivers with "Notch" Filters, Saturated Colours 75% Amplitude

Colour	Correct Value of Relative Luminance	Effective Relative Luminance	
		Double-sideband Receiver	Single-sideband Receiver
Black	0	0	0
Yellow	0·45	0·35	0·38
Red	0·15	0·023	0·05
Green	0·29	0·13	0·17
Blue	0·055	0·0025	0·0073
White	0·75	0·75	0·75
Magenta	0·21	0·053	0·095
Cyan	0·35	0·21	0·28

If the receiver is of the normal monochrome type not incorporating a "notch" filter, both the luminance and chrominance signals are fed to the receiver cathode ray tube. It has been shown[5] that in this case the non-linear brightness/input-voltage characteristic of the tube causes an increase of luminance as a result of rectification of the chrominance signal. The mean brightness of the monochrome picture $\overline{M^{\gamma}}$ over any area of itself represented by the particular values of E'_y and E'_c is given by[5]

$$\overline{M^{\gamma}} = \frac{1}{\pi} \int_{0}^{\pi-\theta_1} \cdot [E'_y + E'_c \cos\theta] \, d\theta \tag{9}$$

where

$$\theta = \omega_s t + \phi$$

$$\phi = -\arctan \frac{E_Q}{E_I}$$

$$\theta_1 = \arccos \frac{E'_y}{E'_c}$$

$$\gamma = 2{\cdot}5$$

Table 5 shows the calculated values of $\overline{M^{\gamma}}$ for saturated colours of 75% amplitude for both double-sideband and single-sideband receivers, and also for true panchromatic representation. Table 5 shows that single-sideband reception is more nearly panchromatic than double-sideband reception.

With the American system, which employs negative modulation, the reduction of the luminance signal will deteriorate the panchromatic scale of the compatible monochrome picture.

TABLE 5

Effective Relative Luminances for Receivers without "Notch" Filters
Saturated Colours 75% Amplitude

Colour	Correct Value of Relative Luminance	Effective Relative Luminance	
		Double-sideband Receiver	Single-sideband Receiver
Black	0	0	0
Yellow	0·45	0·39	0·47
Red	0·15	0·11	0·15
Green	0·29	0·25	0·29
Blue	0·055	0·032	0·042
White	0·75	0·75	0·75
Magenta	0·21	0·15	0·19
Cyan	0·35	0·35	0·42

5. CONCLUSIONS.

The reception of colour transmissions with a vestigial-sideband receiver incorporating an incoherent detector results in distortion of the luminance signal. In the N.T.S.C. system modified to British standards, the result on colour receivers is to reduce the saturation of the colours; on a monochrome receiver the result is to improve the panchromatic scale of the compatible monochrome picture. With the N.T.S.C. system, the result is an increase in the saturation of colours and a deterioration of the panchromatic representation of the compatible monochrome picture.

The above mentioned effects, although readily observable for the 100% full amplitude cases, are not considered to be of a serious nature for the more usual lower amplitudes; in practice, saturated colours may be regarded as exceedingly rare.

6. ACKNOWLEDGMENT.

The curve of a_1 as a function of m in Fig. 2, was computed by R.V. Harvey.

7. REFERENCES.

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